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RAN-2103000206020036

B.Sc. (Sem. VI) Examination April - 2023

Mathematics - MTH-606 - Number Theory - II

સૂચના : / Instructions

(૧) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **B.Sc. (Sem. VI)**

Name of the Subject :

☛ **Mathematics - MTH-606 - Number Theory - II**

Subject Code No.: **2103000206020036**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Follow usual notations.
(4) Figures to the right indicate marks of the question.
(5) Total marks 50.

Que:l Answer any FIVE as directed. [10]

[1] State the necessary and sufficient condition for the solution of the linear congruence $ax \equiv b(mod\ n)$.

[2] If odd prime "p", then prove that
$$1^{p-1} + 2^{p-1} + ... + p^{p-1} \equiv -1\ (mod\ p)$$

[3] Show that $a^{21} \equiv a(mod\ 15)$, for any integer "a".

[4] Prove that $\sum_{k=1}^{n} \mu(k!) = 1$, where $n \geq 3$.

[5] Prove: $\tau(n)$ is an odd integer if and only if "n" is a perfect square.

[6] Prove that $11^{104} + 1$ is divisible by 17.

[7] If "n" is an odd integer, then prove that $\phi\ (2n) = \phi\ (n)$.

[8] If $a \equiv b(mod\ n)$ and $c \equiv d(mod\ n)$, then prove that $ac \equiv bd(mod\ n)$.

Que:2 Attempt any TWO. [10]

[1] The linear congruence $ax \equiv b(mod\ n)$ has a solution. If and only if $d|b$; where $d \equiv gcd(a,n)$. If $d|b$, then it has "d" mutual incongruent solution, two numbers are in when after being divided by the same number.

[2] Find the smallest integer $a > 2$, such that $2|a$, $3|a + 1$, $4|a + 2$, $5|a + 3$ and $6|a + 4$.

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[3] Solve the given system of congruence.

$$x \equiv 3 \ (mod\ 11)$$
$$x \equiv 5 \ (mod\ 19)$$
$$x \equiv 10 \ (mod\ 29)$$

Que:3 Attempt any TWO. [10]

[1] If "p" and "q" are distinct prime, then prove that

$$p^{q-1} + q^{p-1} \equiv 1 (mod\ pq)$$

[2] State and prove Wilson's Theorem.

[3] Verify that $18^6 \equiv 1 (\text{mod } 7^k)$, k = 1, 2, 3, ...

Que:4 Attempt any TWO. [10]

[1] Prove that τ and σ are multiplicative function.

[2] Prove the following:

[a] For $k \geq 2$, $n = 2^{k-1}$ satisfies the equation $\sigma(n) = 2n - 1$.

[b] If 2^{k-1} is prime, then $n = 2^{k-1} (2^k - 1)$ satisfies the equation $\sigma(n) = 2n$.

[3] Find the number of zeros in which 1000! terminates.

Que:5 Attempt any TWO. [10]

[1] If "m" and "n" are relatively prime positive integer, then prove that $m^{\phi(n)} + n^{\phi(m)} \equiv 1 (mod\ mn)$.

[2] If $n = p_1^{k_1} . \ p_2^{k_2} ... \ p_r^{k_r}$ is the prime factorization of the integer $n > 1$, then prove that $\phi(n) = n . \prod_{i=1}^{r} \left(1 - \frac{1}{p_i}\right)$

[3] Using Euler's theorem find the last two digits in the decimal expansion of 3^{256} and 3^{251}.